Newtonian laws derived from energy and information arguments

(cf. Erik Verlinde)

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Abstract

Displacement of a mass gives change in information that costs energy, which results in an attractive force that we experience as gravity.

Deriving Newton's Second Law

Unruh's notion[1] of vacuum can be expressed in an equation giving the equivalent energy $k_B T$ of a uniformly accelerating particle with acceleration a, as:

$$k_B T = \frac{1}{2\pi}\frac{\hbar a}{c} \tag{1}$$

Entropy changes a certain amount ΔS when a particle moves over a distance Δx, therefore an energy difference ΔE occurs. In a closed adiabatic system ΔE is equal to the amount of Work done by the force F that moves the particle, thus according to the laws of thermodynamics:

$$\Delta E = W = F\Delta x = T\Delta S \tag{2}$$

Because of Heisenberg's uncertainty principle, the smallest possible change of distance Δx to consider for a particle of mass m is the Compton wavelength[2]:

$$\Delta x = \frac{\hbar}{mc} \tag{3}$$

This corresponds to the smallest possible entropy change ΔS possible, namely that of moving one bit of information, which is given by:

$$\Delta S = 2\pi k_B \tag{4}$$

Newton's Second Law can be derived using the above formulae (1),(2),(3) and (4), as follows:

$$F = T\frac{\Delta S}{\Delta x} = \frac{1}{2\pi}\frac{\hbar a}{ck_B}\frac{\Delta S}{\Delta x} = \frac{1}{2\pi}\frac{\hbar a}{ck_B}\frac{2\pi k_B}{\frac{\hbar}{mc}} = ma$$

Deriving Newton's Law of Gravity

Consider the number of bits of information (= degrees of freedom) on the surface of a holographic[3] sphere with area A and radius R. This is its area $4\pi R^2$ divided by the space occupied by 1 bit, the Planck area l_P^2 or a "Planckian pixel":

$$\#\text{bits} = \frac{A}{l_P^2} = \frac{4\pi R^2 c^3}{G\hbar} \tag{5}$$

where the Planck length $l_P = \sqrt{\frac{G\hbar}{c^3}}$

According to the equipartition theorem, energy will be divided equally over all bits on the sphere, so the energy per bit is:

$$\frac{1}{2}k_BT = \frac{E}{\#\text{bits}} = \frac{Mc^2}{\#\text{bits}} \tag{6}$$

Using formulae (1),(5) and (6), Newton's Law of Gravity can be derived from Newton's Second Law as follows:

$$F = ma = m\frac{2\pi c}{\hbar}k_BT = m\frac{2\pi c}{\hbar}\frac{2Mc^2}{\#\text{bits}} = m\frac{2\pi c}{\hbar}2Mc^2\frac{G\hbar}{4\pi R^2c^3} = G\frac{mM}{R^2}$$

References

[1] The Canadian physicist William Unruh showed that the notion of vacuum depends on the path of the observer through spacetime. From the viewpoint of the accelerating observer, the vacuum of the inertial observer will look like a state containing many particles in thermal equilibrium, a warm gas.For more information, see `http://en.wikipedia.org/wiki/Unruh_effect`

[2] For a free particle in discrete wave mechanics, the Heisenberg uncertainty principle for position x and momentum p is $\Delta x \geq \frac{\hbar}{\Delta p}$. Because p can never be greater that mc, it follows that the minimum value for Δx is $\frac{\hbar}{mc}$, the Compton wave length.

[3] The holographic principle is a property of quantum gravity and string theories which states that the description of a volume of space can be thought of as encoded on a boundary to the region like a gravitational horizon. In other words, the maximum entropy in a region is bounded by the area of that region. For more information, see `http://en.wikipedia.org/wiki/Holographic_principle`